Two Seemingly Different Beginnings of Numerical Understanding

Grupo de Trabalho*

The work presented here compares two groups of three- and four-year-old children who differ in their school entrance age and consequently in the social expectations of what they should know at these particular ages. The comparison was made in order to assess the effect of children's counting ability on their ability to reason about number. An experiment was designed to make sure that children who succeeded were basing their answers only on the operation performed. The experiment included a set size with more items in it than any of the children would supposedly be able to count. Also, some of the tasks combined addition and subtraction with lengthening and shortening and homogeneous sets were used to avoid the possibility of an answer being based on the presence or absence of an specific item. Three possible outcomes were hypothesised and the results obtained were analysed in the light of these. The results support the conclusion that young children may reason about number even without having represented it and that children's counting ability does not necessarily underlie their capacity to identify number-relevant operations but rather that the ability to make number-based judgements develops independently from the knowledge of counting.

Over the past twenty years or so, research into children's understanding of number has shown how revealing it is to focus on the capabilities young children do have and how important it is to design experiments which allow them to demonstrate their abilities (e.g. Brown and Deloache 1978; Dockrell et al. 1980; Donaldson 1978; Gelman and Gallistel 1978; Light et al. 1979; McGarrigle et al. 1978; Moore and Frye 1986; Trabasso et al. 1978). There is now a general acceptance that pre-school children may have some knowledge about number and counting before they go to school.

Researchers on pre-school children's ability to deal with numbers make a distinction between the ability to represent the numerical value of a given set of elements and the ability to reason about numbers. Research on the former ability involves questions such as whether children can say how many elements are contained in a set and what processes are used to achieve an accurate representation of its numerical value. Research on the latter ability involves questions such as whether children know that the operations of addition and subtraction increase and decrease number; it also involves questions such as whether children know that these operations are related, with one undoing the effect of the other.

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Children have been found to be able to represent small set sizes (e.g. Gelman and Gallistel 1978; Lawson et al. 1974; Smither et al. 1974), to reason about small numbers (e.g. Groen and Resnick 1977; Hughes 1981; Starkey and Gelman 1982) and to be able to count (e.g. Gelman and Gallistel 1978; Saxe 1979; Sophian 1987). Counting seems to be "the" mechanism used by young children to represent numbers and to reason about numbers.

The ability to count implies the ability to follow a set of counting principles, among them those which deal with rules of procedure or the "how-to" of counting: the one-one principle, the stable-ordering principle and the cardinal principle (e.g. Gelman and Meck 1983; Gelman et al. 1986; Greeno et al. 1984).

The one-one principle involves the ticking off of the items in a set with distinct ticks in such a way that one and only one tick is used for each item. This principle requires the co-ordination of two basic component processes: partitioning and tagging. Partitioning involves the step-by-step maintenance of two categories of items: those that have not yet been counted and those that have already been counted. Tagging involves the summoning up, one at a time, of distinct tags. The stable-ordering principle involves the utilisation of a stable ordered list that is as long as the number of elements in a set requires it to be. The cardinal principle involves the fact that the final tag used in a series of tags represents a property of the set as a whole: its cardinal number.

The application of the cardinal principle presupposes the application of the first two principles. Children seem to learn the difference between counting and non-counting words very early (e.g. Fuson et al. 1982). It has been argued (e.g. Gelman and Gallistel 1978) that to demonstrate that a child treats number as invariant is to show that she has a set of rules that classifies the operations which can be performed on a set as being relevant or irrelevant to number. The operations of addition and subtraction are classified as being relevant because they change the original number, while the operations of displacement and rearrangement are classified as irrelevant because they do not change the original number. The abilities to identify that the operations of addition increase number and that subtraction decreases number, that any other transformation, such as displacements of items, leaves number unaltered and yet the understanding that addition and subtraction are related, with one undoing the effect of the other, are considered to be closely related to the ability to produce numbers. Thus, a young child will identify a transformation as numerically relevant or irrelevant if she has the capacity to represent the quantity displayed before and after the change.

It has been proposed that the acquisition of the ability to count skilfully is initially directed by the implicit knowledge of some counting principles (e.g. Gelman and Meck 1986; Greeno et al. 1984). On the other hand, it has also been suggested that children first learn to use numbers mechanically and only gradually induce some counting principles (e.g. Baroody 1984; Baroody and Ginsburg 1986; Briars and Siegler 1984; Fuson 1988; Fuson et al. 1982; Fuson and Hall 1983; Frydman 1995; Wynn 1990). No matter what comes first - principle or skill - children's ability to count will be acquired through their socially organised/structured experiences.

Differential expectations on children's learning at a given age will affect their socially organised experiences and consequently their ability (e.g. Carraher 1986; Saxe et al. 1987). Expectations on children's learning at a given age can be seen for instance at the school entrance age. As such, differential school entrance ages will affect expectations of what children are supposed to know and be able to achieve at a particular age. In this way, differential expectations on children's learning at a given age will affect children's socially organised experiences. Differential socially organised experiences will supposedly affect children's ability to deal with numbers at a particular age.

The work presented here compares two cultures/groups - Group A and Group B - of three- and four-year-old children who differ in their school entrance age and consequently in the expectations of what they should know at these particular ages. Group A children start First School at the age of five years and nurseries for three- and four-year olds generally have a range of number related activities which are viewed as preparatory games for the number work they will do in First School. Group B children do not start First School until the age of seven years and nurseries put little if any emphasis on teaching number skills to three- and four-year-old children. Activities at nurseries for Group B become similar to those of Group A nurseries when the children reach five- and six-years of age. At these ages children are prepared for the number work they will do in First School.

The comparison between these groups was made in order to assess the effect of children's counting ability on their ability to reason about number. An experiment was designed using a procedure where the children had to know that addition/subtraction change number in order to be successful. To make sure that children who succeeded were basing their answers only on the operation performed, the design of the experiment included a set size with more items in it than any of the children would supposedly be able to count (21 elements). Also, some of the tasks combined addition and subtraction with lengthening and shortening; homogeneous sets were used to avoid the possibility of an answer being based on the presence or absence of an specific item. All the tasks were randomised. Three possible outcomes were hypothesised:

1) children whose judgements were based on counting the set should answer all questions correctly for the small set size and be wrong for the larger set size (based on the assumption that they would be unable to count up to 21 elements);

2) children whose judgements were based on a "knowledge" about specific operations should get addition/subtraction operations correct for both the small and large set sizes, even if they were unable to count and even when conflicting perceptual changes were introduced;

3) children whose judgements were based on perceptual changes should succeed only when addition/subtraction were accompanied by an appropriate perceptual change - addition combined with spreading and subtraction combined with condensing.

Method

Subjects

Sixty children took part in this experiment, there being thirty Group A children and thirty Group B children. There were fifteen three-year-old and fifteen four-year-old children in each group. The mean age of Group A children was 3.8 years and 4.5 years, that of Group B children was 3.10 years and 4.9 years. Both groups of children attended nurseries situated in similar residential areas and drew on a population of children from lower-middle to middle socio-economic levels. The aim of the nursery for Group A was to prepare children for their entry into First School; the children were encouraged to engage in a range of number activities, with the nursery staff frequently joining in these games; they also monitored children's progress and kept records of their ability levels. Activities for Group B children did not directly involve number games. Typical activities provided for the children were cutting, painting, gluing, fairly tale stories; in other words, the aim of the nursery for Group B did not involve purposeful preparatory learning for numerical thinking. About one hundred children from two and half to five years of age attended both nurseries. They attended either in the morning or the afternoon on a regular, daily basis. In both, there were classrooms which provided a base for around twenty children with two staff members in charge of them. All children attending both nurseries lived at home with their families (their parents brought them to the nursery and took them home on a daily basis).

Materials

The materials used in this experiment were a set of four different sized cut out representation of teddy bears, a plate and a set of twenty-two green buttons. The buttons were all the same size. Each of the four teddies was wearing a shirt without buttons but showing the buttonholes (teddy A: two buttonholes; teddy B: four buttonholes; teddy C: twenty buttonholes; Teddy D: twenty-two buttonholes).

Procedure

The buttons for each of the four teddies' shirts were put on the plate, not in a row but scattered about the plate to avoid alternative cues to number. The teddy was stood in an upright position behind the plate with buttons on it. Either teddy A or teddy B were presented with the plate containing a set of three buttons and either teddy C or teddy D with the plate containing a set of 21 buttons. The children were shown three or 21 buttons in a plate and a given teddy bear and were told that those buttons were teddy's buttons. No mention of number was made.

When children were presented with teddy A (two buttonholes) and the plate containing three buttons or teddy C (twenty buttonholes) and the plate containing 21 buttons, they were asked to help teddy to decide how to lose a button / to get rid of a button (he had too many buttons). When they were presented with teddy B (four buttonholes) and the plate containing three buttons or teddy D (twenty-two buttonholes) and the plate containing 21 buttons they were asked to help teddy to decide how to get another button (he needed another button). The teddy needed as many buttons as buttonholes in order to be able to button up his shirt.

The following is an example to illustrate the procedure adopted: "...here we have a teddy bear (the child is presented with, for instance, teddy B - four buttonholes) and here we have these buttons (the child is presented with the plate containing three buttons)...he is wearing a shirt with buttonholes in it but without the buttons...he has got these buttons (pointing to the buttons in the plate) for his shirt, but he lost a button...so, he wants another one...now, let us see in which ways we can help teddy to get another button...if we do this (experimenter adds one button to the ones in the plate)...has teddy got another button?...(the child gives his/her answer)...right, in this way we... (experimenter repeats the child's answer). Then the experimenter takes out one button... now, let us try another way...remember, this teddy has got those buttons, but he needs another one...if we do this (experimenter shakes the plate containing the buttons, spreading them out)...has teddy got another button?..." In a similar way, the other changes already discussed were carried out.

Each child was tested on set sizes three and 21 and the experimenter made eight sorts of changes in each of the two set sizes which are discussed below. After each change made on the plate containing the buttons for teddy A and teddy C, children were asked "has teddy lost a button?". On the other hand, after each change made on the plate containing the buttons for teddy B and teddy D, children were asked "has teddy got another button?". The same question was always rephrased in many possible ways without making any mention of arithmetic terms but making sure that the child understood what he/she was required to do.

The question "has teddy got another button?" was asked after the following changes:

1. Addition: one button was added to the set;

2. Spreading: the buttons were spread out by shaking the plate;

3. Addition combined with spreading: one button was added to the set and the buttons were then spread out by shaking the plate;

4. Addition combined with condensing: one button was added to the set and the buttons were put closer to one another.

The question "has teddy lost a button?" was asked after the following changes:

1. Subtraction: one button was taken from the set;

2. Condensing: the buttons were put closer to one another;

3. Subtraction combined with spreading: one button was taken from the set and the rest was spread out by shaking the plate;

4. Subtraction combined with condensing: one button was taken from the set and the buttons were put closer to one another.

The changes for each set size were done in random order. After the game, the children were asked to count a set size containing five items.

RESULTS

The data were analysed to give:

a) the percentage of children who gave correct answers to the question "has teddy got another button?" for each of the two set sizes and changes performed;

b) the percentage of children who gave correct answers to the question "has teddy lost a button?" for each of the two set sizes and changes performed;

c) the percentage of children who were able to count a set containing five items.

- The question "has teddy got another button?":

Table 1 presents the percentage of correct answers for all the conditions.

TABLE 1. PERCENTAGE OF CHILDREN WHO GAVE CORRECT ANSWERS TO THE QUESTION "HAS TEDDY GOT ANOTHER BUTTON?" FOR EACH CHANGE PERFORMED.

	Changes and set sizes							
	addition		addition/spreading		addition/condensing		spreading	
Group /age	3	21	3	21	3	21	3	21
A/3	93	60	93	60	93	47	60	20
A/4	100	93	93	80	93	73	60	53
B/3	100	100	100	100	87	87	73	73
B/4	100	100	100	100	100	100	100	100

a) ADDITION

The data show that Group B children in both age groups and Group A four-year-olds consistently gave correct responses for both set sizes when a button was added to those on the plate. Group A three-year-olds consistently answered correctly for set size three and 60% of them were correct for set size 21. There was no set size effect for the Group B children. The only significant difference in performance was between the Group A three-year-old and the Group B three-year-old children when dealing with set size 21 (χ^2=7.5, df=1, p<0.01).

b) ADDITION/SPREADING

The data show that the Group B children in both age groups and the Group A four-year-old children consistently gave correct responses for both set sizes. On the other hand, the Group A three-year-old children performed better for set size three, with 60% of them answering correctly for set size 21. Again, set size effect appeared only among the Group A children, affecting mainly the three-year-olds' performance. Once more, the only significant difference in performance between Group A and Group B was that between the three-year-old children for set size 21 (χ^2=7.5, df=1, p<0.01).

c) Addition/Condensing

The data show that Group B children in both age groups consistently answered correctly for both set sizes and the Group A children in both age groups consistently answered correctly for set size three, with 47% of the three-year-olds and 73% of the four-year-olds answering correctly for set size 21. The set size effect remains only for the Group A children. The difference in performance between the Group A and the Group B three-year-old children (χ^2=5.4, df=1, p<0.05) and also between the Group A and the Group B four-year-old children (χ^2=4.61, df=1, p<0.05) were both found to be significant in relation to set size 21.

d) Spreading

For this transformation only the Group B four-year-olds consistently gave correct responses, with 73% correct answers from the three-year-olds. The Group A children in both age groups were more successful with set size three (60% correct answers). With set size 21, 53% of the four-year-olds and only 20% of the three-year-olds answered correctly. Again, the set size effect appears only for the Group A children. There was a significant difference in performance between the Group A and the Group B three-year-olds in relation to set size 21 (χ^2=8.57, df=1, p<0.01), and between the Group A and the Group B four-year-olds for both set size three (χ^2=7.5, df=1, p<0.01) and 21 (χ^2=9.13, df=1, p<0.01).

Both groups showed a decrease in correct responding in the tasks involving spreading and addition/condensing changes. Apart from the Group B four-year-old children, all the other groups showed a deterioration in performance when spreading only was carried out. When comparing, for example, the performance between spreading only and addition for a set as small as three elements, the drop in correct responses was found to be significant for the Group A three-year-olds (χ^2=4.66, df=1, p<0.05), the Group B three-year-olds (χ^2=4.62, df=1, p<0.05) and even for the Group A four-year-olds (χ^2=7.5, df=1, p<0.01).

- The question "has teddy lost a button?":

Table 2 presents the percentage of correct answers for all the children.

TABLE 2. PERCENTAGE OF CHILDREN WHO GAVE CORRECT ANSWERS TO THE QUESTION "HAS TEDDY LOST A BUTTON?" FOR EACH CHANGE PERFORMED

	Changes and set sizes							
	subtraction		subtraction/condensing		subtraction/spreading		condensing	
Group /age	3	21	3	21	3	21	3	21
A/3	87	60	93	60	93	47	87	53
A/4	100	100	100	100	93	93	87	80
B/3	100	100	100	100	87	87	93	93
B/4	100	100	100	100	100	100	100	100

a) SUBTRACTION

The data show that all the Group B children in both age groups and the Group A four-year-olds consistently answered correctly for both set sizes. The Group A three-year-olds consistently answered correctly for set size three, with 60% correct answers for set size 21. A set size effect can be seen only among the Group A three-year-olds. A significant difference in performance was found only between the Group A and Group B three-year-old children for set size 21 (χ^2=7.5, df=1, p<0.01).

b) SUBTRACTION/CONDENSING

The data show that again Group B children in both age groups and the Group A four-year-olds consistently answered correctly for both set sizes, while the Group A three-year-olds were consistently correct for set size three with 60% correct answers for set size 21. Again, a set size effect appears only with the Group A three-year-olds. The difference in performance between the Group A and the Group B three-year-old children for set size 21 was found to be significant at the 1 percent level (χ^2=7.5, df=1, p<0.01).

c) SUBTRACTION/SPREADING

Again, Group B children and Group A four-year-olds for both set sizes and the Group A three-year-olds for set size three consistently answered correctly, with 47% correct answers for set size 21. There is a clear set size effect in the performance of the Group A three-year-olds. The only significant difference in performance was between the Group A and the

Group B three-year-old children when dealing with set size 21 (χ^2=5.7, df=1, p<0.02).

d) Condensing

All the children consistently answered correctly for both set sizes except for the Group A three-year-olds for set size 21 (53% correct answers). A set size effect can be seen among the Group A children. Once again, when dealing with set size 21, the difference in performance between Group A and Group B three-year-olds was found to be significant (χ^2=6.14, df=1, p<0.02).

- The percentage of children who counted correctly set size five:

The data (Table 3) show that while Group A children in both age groups consistently counted correctly up to five, however, only a very small percentage of the Group B three- and four-year-olds, that is, 13% and 20% respectively, were able to represent this quantity.

TABLE 3. PERCENTAGE OF CHILDREN WHO COUNTED A SET SIZE OF FIVE ITEMS CORRECTLY

	Age (years)	
Group	three	four
A	80	87
B	13	20

Discussion

The results obtained were analysed in the light of the three outcomes hypothesised in the introduction. The first one was that if children were to base their judgements on counting the set, they should get all questions correct for the small set size and all wrong for the large set size (based on the assumption that they would be unable to count to 21) no matter what change was performed. There is a very strong indication that counting was not the mechanism used by the Group B children. Only 13% of the Group B three-year-olds (two children) and 20% of the four-year-olds (three children) were able to count up to five, yet at least 73% of the three-year-old and 100% of the four-year-old children succeeded in the tasks. Moreover, there was no set size effect, and only a small age effect. This supports the conclusion that they were not relying on a counting process, because if they were, this effect would appear since it is known that as set size increases the children's performance decreases.

On the other hand, Group A four-year-old children performed better than the three-year-olds. More than 80% of the four-year-olds gave correct answers for most of the changes except for spreading in both set sizes and addition/condensing for set size 21. It is interesting to observe that more than 80% of the children in both age groups were successful when dealing with set size three except in the task where only spreading was carried out. There is a clear set size effect on Group A children's performance. Their performance suggests that they could be basing their judgements on counting the set. Even when addition or subtraction was accompanied by conflicting perceptual changes the drop occurred in relation to set size 21 and mainly among the three-year-olds.

The second prediction was that if children were to base their judgements on a "knowledge" about specific operations they should get relevant "operations" (addition and subtraction) correct, no matter what they were combined with, for both the small and large set sizes (even when unable to count). All the Group B four-year-olds and at least 87% of the three-year-olds considered addition and subtraction, not only when performed alone but also when combined with both appropriate and conflicting perceptual changes, as number-relevant operations. So, it seems that they were sensitive to operations that are numerically relevant even without being able to count. A considerable percentage of the Group A four-year-olds (more than 90% when dealing with the question "has teddy lost" and from 73% up to a ceiling of 100% when dealing with the question "has teddy got") considered addition and subtraction combined with both sorts of perceptual changes as number-relevant operations. The performance of the three-year-olds clearly showed that they were more successful when dealing with set size three.

The third outcome hypothesised was that if children were to base their judgements on perceptual changes they should succeed only when addition and subtraction were accompanied by an appropriate perceptual change, that is, addition/spreading and subtraction/condensing. The results showed that 100% of the Group B four-year-olds succeeded no matter what change was performed. The three-year-olds showed a drop in correct responding when spreading, condensing, addition/condensing and subtraction/spreading changes were performed. Thus, some of the three-year-olds who were correct for addition, subtraction, addition/spreading and subtraction/condensing were in fact basing their answers on perceptual changes, taking into account that when addition/condensing and subtraction/spreading - conflicting perceptual changes - were executed they failed. However, those children represented only 13% of the group. The results support the conclusion that most Group B children were not basing their judgement on counting nor on perceptual changes.

The performance of the Group A four-year-olds showed a small decrease in the tasks involving addition/condensing and subtraction/spreading. Although some of them may have been responding to perceptual cues, a high percentage of them succeeded in the task. Those children who failed when addition or subtraction were accompanied by conflicting cues represented only 7% of the group who were correct for subtraction/condensing for both set sizes and addition/spreading for set size 21. For the three-year-olds there was a drop in correct responding when the operations on set size 21 were combined with conflicting perceptual cues. This drop represents 13% of the group. Spreading or condensing alone created much more confusion than addition or subtraction combined with any of the perceptual changes.

The fact that most of the Group B three-year-old and the Group A four-year-old children were more successful when addition or subtraction occurred, alone or combined with displacements, than when spreading or condensing alone were carried out would suggest that they were taking into account the operation itself. In other words, the act of putting in a new element or taking away a present one was strong enough to make the children, in some way, to ignore the perceptual changes that took place together with the operations. These results suggests that Group A four-year-old children could be basing their answers on counting the sets but they were also considering the operation itself. So, it seems that they were also sensitive to numerically relevant operations.

It seems that pre-school children are able to reason about numbers even before being able to count. Group A and Group B children performed statistically at the same level most of the time. When the difference was significant, it was always "in favour" of Group B children, with the Group B three-year-old performing better than Group A three-year-old children when set size 21 was involved.

The results support the conclusion that young children may reason about number even without having represented it. A high percentage of Group B children, most of them unable to count, seemed to be able to recognise that the operations of "adding to/subtracting from" do change the number of elements contained in a set.

Children's counting ability did not seem to have a particular or significant effect on their ability to reason about number. What explanation can be offered for these results? Perhaps counting and the understanding of number are not as closely tied together (e.g. Siegler 1991; Sophian 1988) as some of the other researchers have supposed. Perhaps it would indeed be essential to make a distinction between the concept of number and concepts of counting as suggested by Frydman (1995) since the results seem to support the conclusion that children's counting ability does not necessarily underlie their capacity to identify number-relevant operations but rather that the ability to make number-based judgements develops independently from the knowledge of counting.

On the other hand, the way by which children develop the ability to reason about number may in fact vary as a function of the context in which they encounter number. In this way, different experiences will affect not only children's ability to deal with numbers at a particular age but also the particular way through which they will deal with it (e.g. Saxe et al. 1987; Sophian 1995). In this particular case, the developmental context of the children was comparable in many respects. The two groups - Group A and Group B - of children were exposed to an urban environment reflecting similar contact with modern, technological industrial living, to similar socio-economic level, attending nursery school, and because they differed at the school entrance age, they automatically differed in the adults' interest in teaching number names and counting games to three- and four-year-old children.

Group A children were exposed to a wide range of activities which gave them practice in matching sets, categorising, counting, judgement of equivalents, etc., and many number skills were emphasised in pre-school education for three- and four-year-olds. The relationship between their counting ability and their ability to reason about number was undoubtedly determined by these socially organised experiences which explicitly trained them to deal with number in a particular way. Consequently, when encountering any numerical situation they tended to immediately apply those formal tools that were provided to them. They recognise it as the right way through which such situations should be dealt with.

Group B children were exposed to a context where there were not formal educational goals to attain in nurseries for three- and four-year-old children, consequently they were not formally trained to acquire number skills at an early age. However, they encountered number in many daily activities and probably had to construct a way to deal with it. Even though most of them could not count, they showed an unexpected ability to reason about number. Even without formal training they encountered many natural numerical situations in their daily activities, in their day-to-day play, and probably ended up developing particular strategies in trying to deal with such situations. The relationship between their counting ability and their ability to reason about number was also undoubtedly determined by these informal encounters with numerical situations which in a certain way also trained them to deal with number in a particular form.

Differences in the situations in which number is practised will lead to differences in the way children deal with numerical concepts (e.g. Carraher et al. 1985; Carraher et al. 1987). However, the counting ability may probably be more closely related to children's reasoning about the reversibility of number-relevant operations than has been supposed (e.g. Ashcraft 1982; Fuson 1982; Woods et al. 1975).

REFERENCES

ASHCRAFT, M.H. "The development of mental arithmetic: a chronometric approach", *Developmental Review*, 2, (1982): pp.213-236.

BAROODY, A.J. "More precisely defining and measuring the order-irrelevance principle", *Journal of Experimental Child Psychology*, 38, (1984): pp.33-41.

BAROODY, A.J. and GINSBURG, H.P. "The relationship between initial and mechanical knowledge of arithmetic". In HIEBERT, J., ed. *Conceptual and Procedural Knowledge: The Case of Mathematics.* Hillsdale, N.J.: Lawrence Erlbaum Associates,1986.

BRIARS, D. and SIEGLER, R.S. "A featural analysis of pre-schoolers' counting knowledge", *Developmental Psychology*, 20, (1984): pp.607-618.

BROWN, A.L. and DELOACHE, J.S. "Skills, plans and self-regulation". In SIEGLER, R.S., ed. *Children's Thinking: What Develops?* Hillsdale, N.J.: Lawrence Erlbaum Associates, 1978.

CARRAHER, T.N. "From drawings to buildings; working with mathematical scales", *International Journal of Behavioural Development*, 9, (1986): pp.527-544.

CARRAHER, T.N., CARRAHER, D.W. and SCHLIEMANN, A.D. "Mathematics in the streets and in schools", *British Journal of Developmental Psychology*, 3, (1985): pp.21-29.

CARRAHER, T.N., CARRAHER, D.W. and SCHLIEMANN, A.D. "Written and oral mathematics", *Journal for Research on Mathematics Education,* 18, 2, (1987): pp.83.97.

DOCKRELL, J., CAMPBELL, R. and NEILSON, I. "Conservation accidents revisited", *International Journal of Behavioural Development*, 3, (1980): pp.423-439.

DONALDSON, M. *Children's Minds*. Glasglow: Fontana, 1978.

FRYDMAN, O. "The concept of number and the acquisition of counting concepts: the 'when', the 'how', and the 'what' of it", *Current Psychology of Cognition*, 14, *6*, (1995): pp.653-684.

FUSON, K.C. "An analysis of the counting-on solution procedure in addition". In CARPENTER, T.P., MOSER, J.M. and T.A. ROMBERG, eds. *Addition and Subtraction: A Cognitive Perspective.* Hillsdale: NJ: Erlbaum, 1982.

FUSON, K.C. *Children's Counting and Concepts of Number*. Springer Series in Cognitive Development, 1988.

FUSON, K.C. and HALL, J.W. "The acquisition of early number word meanings: a conceptual analysis and review". In GINSBURG, H.P. ed. *The Development of Mathematical Thinking.* New York: Academic Press, 1983.

FUSON, K.C., RICHARDS, J. and BRIARS, D.J. "The acquisition and elaboration of the number word sequence". In BRAINERD, C.J. ed. *Children's Logical and Mathematical Cognition: Progress in Cognitive Development Research.* New York: Spring-Velag, 1982.

GELMAN, R. and GALLISTEL, C.R. *The Child's Understanding of Number.* Cambridge, MA: Harvard Press, 1978.

GELMAN, R. and MECK, E. "Pre-schoolers' counting: principles before skills", *Cognition*, 13, (1983): pp.343-359.

GELMAN, R. and MECK, E. "The notion of principle: the case of counting". In HIEBERT, J. ed. *The Relationship between Procedural and Conceptual Competence*. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1986.

GELMAN, R., MECK, E. and MERKIN, S. "Young children's numerical competence", *Cognitive Development*, 1, (1986): pp.1-29.

GREENO, J.G., RILEY, M.S. and GELMAN, R. "Conceptual competence and children's counting", *Cognitive Psychology*, 16, (1984): pp.94-134.

GROEN, G. and RESNICK, L.B. "Can pre-school children invent addition algorithms?", *Journal of Educational Psychology*, 69, (1977): pp.645-652.

HUGHES, M. "Can pre-school children add and subtract?", *Educational Psychology*, 1, (1981): pp.207-219.

LAWSON, G., BARON, J. and SIEGEL, L. "The role of number and length cues in children's quantitative judgements", *Child Development*, 45, (1974): pp.731-736.

LIGHT, P.H., BUCKINGHAM, N. and ROBBINS, A.H. "The conservation task as an interactional setting", *British Journal of Educational Psychology*, 49, (1979): pp.304-310.

MCGARRIGLE, J., GRIEVE, R. and HUGHES, M. "Interpreting inclusion: a contribution to the study of the child's cognitive and linguistic development", *Journal of Experimental Child Psychology*, 26, (1978): pp.528-550.

MOORE, C. and FRYE, D. "The effect of experimenter's intention on the child's understanding of conservation", *Cognition*, 22, (1986): pp.283-298.

SAXE, G.B. "Developmental relations between notational counting and number conservation", *Child Development*, 50, (1979): pp.180-187.

SAXE, G.B., GUBERMAN, S.R. and GEARHART, M. "Several processes in early number development", *Monograph of the Society for Research in Child Development*, 216, 52, 2, (1987): pp.-.

SIEGLER, R.S. *Children's Thinking*. 2nded. Englewood Cliffs: N.J.: Prentice-Hall, 1991.

SMITHER, S.J., SMILEY, S.S. and REES, R. "The use of perceptual cues for number judgements by young children", *Child Development*, 45, (1974): pp.693-699.

SOPHIAN, C. "Early developments in children's use of counting to solve quantitative problems", *Cognition and Instruction*, 4, (1987): pp.61-90.

SOPHIAN, C. "Early developments in children's understanding of number: inferences about numerosity and one-to-one correspondence", *Child Development*, 59, (1988): pp.1397-1414.

SOPHIAN, C. "The trouble with competence models", *Current Psychology of Cognition*, 14, 6, (1995): pp.753-759.

STARKEY, P. and GELMAN, R. "The development of addition and subtraction abilities prior to schooling in arithmetic". In CARPENTER, T.P., MOSER, J.M. and T.A. ROMBERG, eds. *Addition and Subtraction: A Cognitive Perspective*. Hillsdale: NJ: Erlbaum, 1982.

TRABASSO, T., ISEN, A.M., DOLECKI, P., MCLANAHAN, A.G., RILEY, C.A. and TUCKER, T. "How do children solve class-inclusion problems?" In SIEGLER, R. ed. *Children's Thinking: What Develops?* Hillsdale, N.J.: Lawrence Erlbaum Associates, 1978.

WOODS, S.S., RESNICK, L.B. and GROEN, G.J. "An experimental test of five process models for subtraction", *Journal of Educational Psychology*, 67, (1975): pp.17-21.

WYNN, K. "Children's understanding of counting", *Cognition*, 36, (1990): pp.155-193.